Notes on the Results of the 1997 $K_L \to \pi^+\pi^-\gamma$ Analysis

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References

[1] John Shields. *The Search for the Emission of a CP–Violating E1 Photon in the $K_L \to \pi^+\pi^-\gamma$ Decay*. PhD thesis, University of Virginia, 2004.

[2] L.M. Sehgal and J. van Leusen. Violation of time reversal invariance in the decays $K_L \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^-e^+e^-$ ˙*Physical Review Letters*, **83**:4933, 1999.

1 Introduction

This note describes the calculation of the derived parameters $DE/(DE + IB)$ and $\langle|g_{M1}|\rangle$ using the results of the 1997 $K_L \to \pi^+\pi^-\gamma$ analysis [1]. A value for r is also calculated.

2 Derived Parameters for $K_L \to \pi^+\pi^-\gamma$

While the analysis of the 1997 $K_L \to \pi^+\pi^-\gamma$ data used the parameterization taken from [2], previous analyses used different formalisms for expressing both the absolute and relative strength of M1 direct photon emission. In order for direct comparisons to be made, it is necessary to compute these other parameters.

Using the same model as that used in the analysis of the 1997 $K_L \to \pi^+\pi^-\gamma$ data, it is possible to calculate the parameters $\langle|g_{M1}|\rangle$, $DE/(DE+IB)$ and r directly from the partial widths and matrix elements that describe this decay. All calculations are based on the model of [2] and are valid for 20MeV $< E_\gamma < (m_K^2 - 4m_\pi^2)\,/\,(2m_K)$ —the range used during the $K_L \to \pi^+\pi^-\gamma$ analysis.

The parameters found as a result of the 1997 $K_L \to \pi^+\pi^-\gamma$ analysis

$$\begin{aligned}\widetilde{g_{M1}} &= 1.229 \pm 0.035 \pm 0.087 \\ a_1/a_2 &= -0.733 \pm 0.007 \pm 0.014 \\ |g_{E1}| &< 0.14(90\%\ CL)\end{aligned}$$

were used for these computations, with the exception of $|g_{E1}|$, which was set equal to the best fit value of zero. Before the calculations can begin, it is necessary to first determine the correlation between the errors in $\widetilde{g_{M1}}$ and a_1/a_2 . This was done by assuming that possible values of $\widetilde{g_{M1}}$ and a_1/a_2 are distributed according to a 2-D Gaussian of the form:

$$\begin{aligned} &P\left(\widetilde{g_{M1}}, a_1/a_2, \overline{\widetilde{g_{M1}}}, \overline{a_1/a_2}, \sigma_{\widetilde{g_{M1}}}, \sigma_{a_1/a_2}, \rho\right) = \\ &\quad \frac{1}{2\pi\sigma_{\widetilde{g_{M1}}}\sigma_{a_1/a_2}\sqrt{1-\rho^2}} \\ &\quad \times\ \exp\left\{\frac{-1}{2\left(1-\rho^2\right)}\left(\frac{\left(\widetilde{g_{M1}}-\overline{\widetilde{g_{M1}}}\right)^2}{\sigma^2_{\widetilde{g_{M1}}}} - \frac{2\rho\left(\widetilde{g_{M1}}-\overline{\widetilde{g_{M1}}}\right)\left(a_1/a_2-\overline{a_1/a_2}\right)}{\sigma_{\widetilde{g_{M1}}}\sigma_{aoat}} + \frac{\left(a_1/a_2-\overline{a_1/a_2}\right)^2}{\sigma^2_{a_1/a_2}}\right)\right\}\end{aligned} \tag{1}$$

where the mean values are taken from the measured central values, the variances are taken from the measured statistical errors only, and the correlation parameter ρ is to be determined. A plot was then generated to form a 1σ contour, which was then compared to Figure 8.1 of [1], the 2-D projection of the 3-D error ellipsoid. The value of ρ was adjusted until the generated 1σ contour coincided with that shown in [1]. The value of ρ was determined to be approximately 0.993.

With ρ determined, pairs of values of $\widetilde{g_{M1}}$ and a_1/a_2 were generated within 10σ of their central values, and the pair was then weighted with Eq.1, the errors of which were modified to contain the systematic and statistical errors added in quadrature.

After weighting the pairs, each is used to calculate the derived parameters using the matrix elements from [2], thus building a distribution of values for $\langle|g_{M1}|\rangle$, $DE/(DE+IB)$ and r. Once sufficient statistics are achieved, the central value is chosen as the most probable value of the distribution. The 1σ bounds are taken as the two points of equal probability between which 68% of the area under the probability distribution resides.

2.1 $\langle|g_{M1}|\rangle$

While the 97 $K_L \to \pi^+\pi^-\gamma$ analysis assumed a form factor based on Vector Meson Dominance:

$$FF(E_\gamma) = \left[1 + \frac{a_1/a_2}{M_\rho^2 - M_K^2 + 2M_K E_\gamma}\right]$$

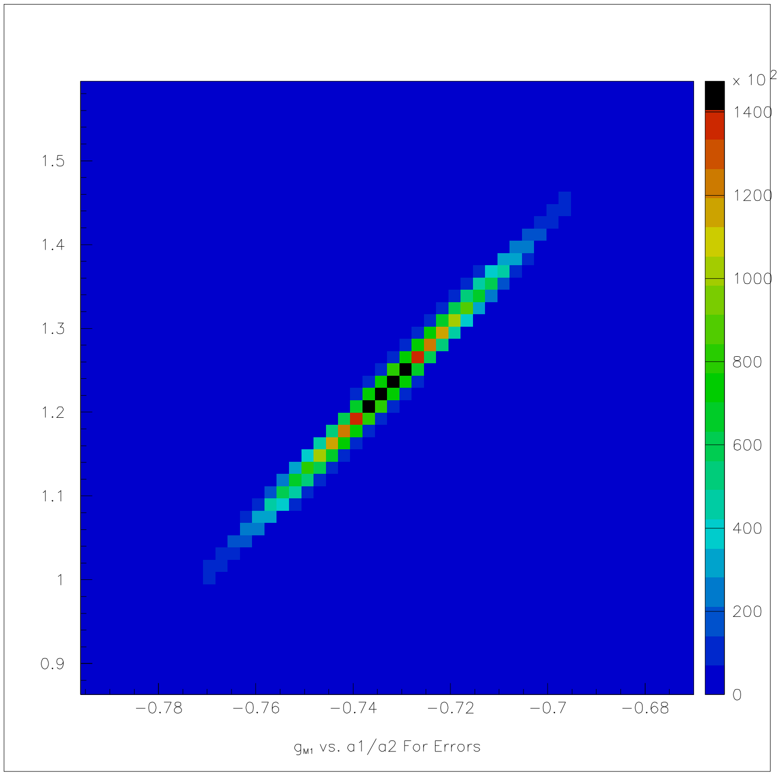

Figure 1: Distribution Of $\widetilde{g_{M1}}$ And a_1/a_2 Used In This Study.

it is useful to calculate the magnitude of the entire M1 amplitude averaged over the region of phase space studied. We do this by noting that the partial width of the M1 emission in $K_L \to \pi^+\pi^-\gamma$ can be written as

$$\begin{aligned} \Gamma^{M1}_{K_L \to \pi^+\pi^-\gamma} &= \int d[PS]\, |M1_{direct}|^2 \\ &= \int d[PS]\, |\widetilde{g_{M1}}|^2 \left(1 + \frac{a_1/a_2}{M_\rho^2 - M_K^2 + 2M_K E_\gamma}\right)^2 \end{aligned} \tag{2}$$

where

$$d[PS] = d\cos\theta\ dE_\gamma\ \sin^2\theta \left(\frac{E_\gamma\sqrt{1-\frac{4M_\pi^2}{M_K^2-2E_\gamma M_K}}}{8\pi M_K}\right)^3 \left(1-\frac{2E_\gamma}{M_K}\right)$$

However, if we assume that the M1 amplitude is independent of photon energy, then we can instead write

$$\Gamma^{M1}_{K_L \to \pi^+\pi^-\gamma} = \int d[PS] \langle |g_{M1}| \rangle^2 \tag{3}$$

where $\langle|g_{M1}|\rangle$ is the average value of the M1 Direct Emission amplitude. If $\langle|g_{M1}|\rangle$ truly is the average value, then the partial widths given by Eq.2 and Eq.3 should be equal, leading to

$$|\langle g_{M1}\rangle| = \sqrt{\frac{\int d[PS]\,|\widetilde{g_{M1}}|^2\left(1+\frac{a_1/a_2}{M_\rho^2-M_K^2+2M_K E_\gamma}\right)^2}{\int d[PS]}}$$

Now by using the best fit values for $\widetilde{g_{M1}}$ and a_1/a_2 , along with the correlation between these parameters, we can now generate a distribution (see Figure 2) of values for $\langle|g_{M1}|\rangle$ from which we can extract a value and error, resulting in a value of:

$$\langle|g_{M1}|\rangle = 0.79\pm^{0.01}_{0.02}$$

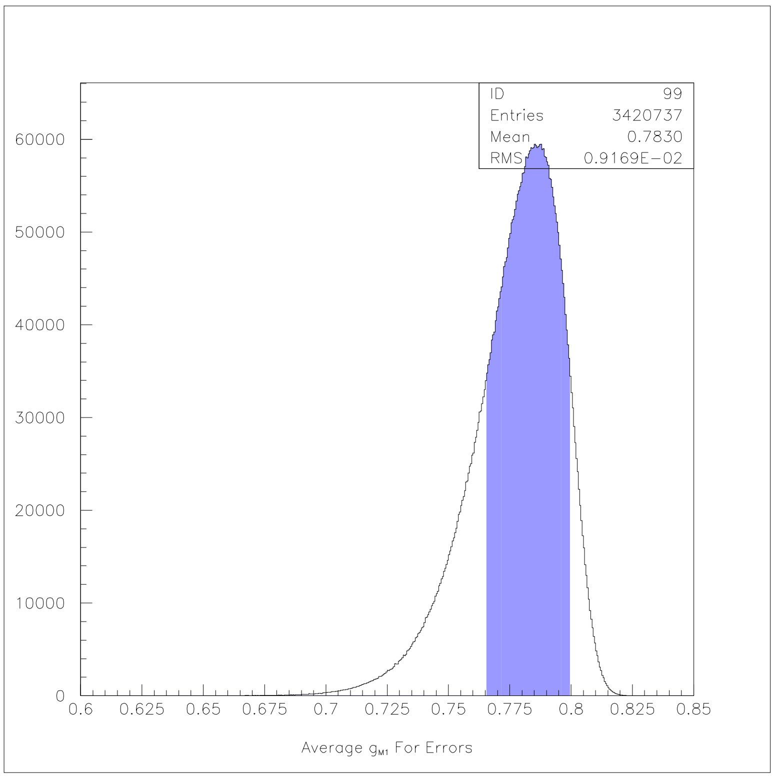

Figure 2: Value of $\langle|g_{M1}|\rangle$ over a range of possible $\widetilde{g_{M1}}$ and a_1/a_2. The area corresponding to the error bounds is shown in blue.

2.2 r

The parameter r, which is defined as:

$$r = \frac{\Gamma^{M1}_{K_L\to\pi^+\pi^-\gamma}}{\Gamma^{E1}_{K_L\to\pi^+\pi^-\gamma}}$$

$$\approx \frac{\int d[PS]\, |\widetilde{g_{M1}}|^2 \left(1 + \frac{a_1/a_2}{M_\rho^2 - M_K^2 + 2M_K E_\gamma}\right)^2}{\int d[PS] \left(\frac{2M_K}{E_\gamma}\right)^4 \frac{|\eta_{+-}|^2}{\left(1 - \left(1 - \frac{4M_\pi^2}{M_K^2 - 2E_\gamma M_K}\right)\cos^2\theta\right)^2}}$$

appears in the decay rate for $K_{L,S} \to \pi^+\pi^-\gamma$

$$\begin{aligned} \frac{dN}{d\tau} \;=\; & \frac{N_K B_{K_S \to \pi^+\pi^-\gamma}}{\tau_S} \Big[|\rho|^2 e^{-\tau/\tau_S} + (1+r)\,|\eta_{+-\gamma}|^2 e^{-\tau/\tau_L} \\ & + 2\,|\rho|\,|\eta_{+-\gamma}| \cos\left(\Delta m_K \tau + \phi_\rho - \phi_{+-\gamma}\right) e^{-(1/\tau_L + 1/\tau_S)\tau/2} \Big] \end{aligned}$$

which is used to extract $\eta_{+-\gamma}$ from $\pi^+\pi^-\gamma$ decays in the regenerator beam. After generating the correct distribution of values for $\widetilde{g_{M1}}$ and a_1/a_2 the result is (see Figure 3):

$$r = 2.31\pm^{0.07}_{0.13}$$

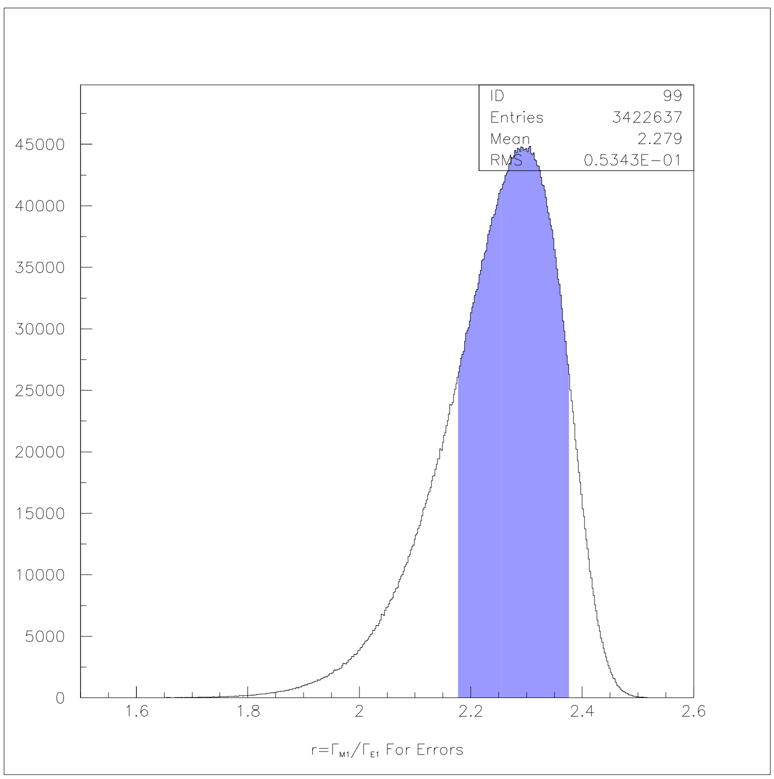

Figure 3: Value of r over a range of possible $\widetilde{g_{M1}}$ and a_1/a_2.

2.3 DE/(DE+IB)

This value is simply an expression of the size of the M1 process in relation to the total process $K_L \to \pi^+\pi^-\gamma$. It is related to r above by:

$$\frac{DE}{DE+IB} = \frac{r}{1+r}$$

when E1 Direct Emission is neglected, as it is here. The result is (see Figure (4)):

$$\frac{DE}{DE+IB} = 0.698\pm^{0.007}_{0.012}$$

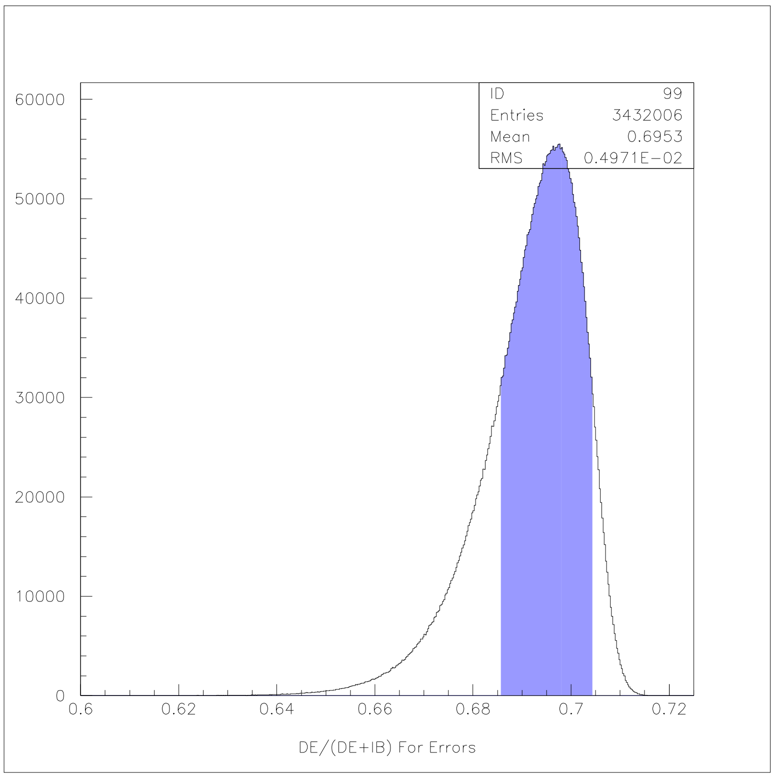

Figure 4: Value of DE/(DE+IB) over a range of possible $\widetilde{g_{M1}}$ and a_1/a_2.